SOLUTIONS TO IIT–JEE 2004 (MAINS)

MEMORY BASED QUESTIONS

MATHEMATICS

1. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are distinct vectors satisfying relation $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$. Prove that $\vec{a}.\vec{b}+\vec{c}.\vec{d} \neq \vec{a}.\vec{c}+\vec{b}.\vec{d}$.

[2]

Sol. Here $\left(\vec{a}-\vec{d}\right)\times\left(\vec{c}-\vec{b}\right) = \left(\vec{a}\times\vec{c}\right)+\left(\vec{d}\times\vec{b}\right)-\left(\vec{a}\times\vec{b}\right)-\left(\vec{d}\times\vec{c}\right) = 0$

$\Rightarrow \quad \left(\vec{a}-\vec{d}\right) \parallel \left(\vec{c}-\vec{b}\right)$

$\therefore \quad \left(\vec{a}-\vec{d}\right).\left(\vec{c}-\vec{b}\right) \neq 0$

$\Rightarrow \quad \vec{a}.\vec{b}+\vec{c}.\vec{d} \neq \vec{a}.\vec{c}+\vec{b}.\vec{d}$

2. If $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$. Find centre and radius of the circle $\left|\dfrac{(z-\alpha)}{(z-\beta)}\right| = k$ where $z = x + iy$.

[2]

Sol. Here

$$|z-\alpha|^2 = k^2\,|z-\beta|^2$$

$$\Rightarrow \quad (z-\alpha)(\bar{z}-\bar{\alpha}) = k^2(z-\beta)(\bar{z}-\bar{\beta})$$

$$\Rightarrow \quad (k^2-1)z\bar{z} + z\left(\bar{\alpha}-k^2\bar{\beta}\right) + \bar{z}\left(\alpha-k^2\beta\right) + \left(k^2.|\beta|^2 - |\alpha|^2\right) = 0$$

Centre $\equiv \dfrac{k^2\beta-\alpha}{k^2-1}$

Radius $= \dfrac{1}{(k^2-1)}\sqrt{|\alpha-k^2\beta|^2 - \left(k^2.|\beta|^2-|\alpha|^2\right).(k^2-1)}$

3. ABCD is the base of parallelopiped T and A′B′C′D′ be the upper face. The parallelopiped is compressed so that the vertex A′ shifts to A″ on a parallelopiped S. If volume of new parallelopiped S is 90% of the parallelopiped T. Prove that locus of A″ is a plane.

[2]

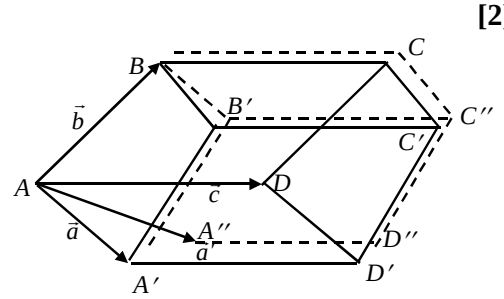
Sol. $\vec{a}.\left(\vec{b}\times\vec{c}\right)\times\dfrac{90}{100} = \vec{a}'.\left(\vec{b}\times\vec{c}\right)$

$\Rightarrow \quad \left(0.9\vec{a}-\vec{a}'\right).\left(\vec{b}\times\vec{c}\right) = 0$

so locus of A″is

$\left(0.9\vec{a}-\vec{r}\right).\left(\vec{b}\times\vec{c}\right) = 0$

Which is a plane.



4. $f(x)$ is defined as $f:(-1,\,1)\to R$ and is differentiable on (–1, 1). It is given that $f'(0) = \lim\limits_{n\to\infty} n\left(f\left(\dfrac{1}{n}\right)\right)$ also $f(0) = 0$. Find the value of $\lim\limits_{n\to\infty}\left(\dfrac{2}{\pi}(n+1)\cos^{-1}\dfrac{1}{n} - n\right)$ given that $\left|\cos^{-1}\left(\dfrac{1}{n}\right)\right| \le \dfrac{\pi}{2}$.

[2]

Sol. $\lim\limits_{n\to\infty}\left(\dfrac{2}{\pi}(n+1).\cos^{-1}\left(\dfrac{1}{n}\right) - n\right)$

$$\Rightarrow \quad \lim_{n\to\infty}\left(\frac{2}{\pi}(n+1)\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{n}\right)-n\right)$$

$$\Rightarrow \quad \lim_{n\to\infty}\left((n+1)-\frac{2}{\pi}(n+1)\sin^{-1}\frac{1}{n}-n\right)$$

$$\Rightarrow \quad \lim_{n\to\infty}\left(1-n.\frac{2}{\pi}\sin^{-1}\frac{1}{n}-\frac{2}{\pi}\sin^{-1}\frac{1}{n}\right)$$

$$= 1-\frac{2}{\pi}.f'(0)-\frac{2}{\pi}\lim_{n\to\infty}\sin^{-1}\frac{1}{n} \qquad [\because \lim_{n\to\infty} n.f\left(\frac{1}{n}\right) = f'(0)$$

Taking $f(n)=\sin^{-1} n$]

$$= 1-\frac{2}{\pi}.f'(0) = 1-\frac{2}{\pi}$$

here $f(x)=\sin^{-1} x$

$$f'(x)=\frac{1}{\sqrt{1-x^2}}$$

$$f'(0)=1$$

5. Using permutation or otherwise, prove that $\frac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer.

[2]

Sol. Let r +1, r + 2 …. r + n are n consecutive integers

Product of these = (r + 1) (r + 2) … (r + n)

$$= \frac{1...r.(r+1)....(r+n)}{1...r}$$

$$= \frac{(n+r)!}{r!} = {}^{n+r}P_n$$

Which is an integer

$$n^2! = (1\times 2.....n)\times((n+1)....2n)\times((2n+1)....3n).....((n^2-n+1)....n^2)$$

There are n groups of n consecutive integers and each will be divisible by n!

So $n^2!$ is divisible by $(n!)^n$.

6. A and B are independent events and C is an event defined such that exactly one of A or B occurs. Prove that $P(C)\geq P(A\cup B).P(\overline{A}\cap\overline{B})$.

[2]

Sol. Here $P(C)=P(A)P(\overline{B})+P(B)P(\overline{A})$

and $P(\overline{A}\cap\overline{B})=P(\overline{A})P(\overline{B})$ [events are independent]

$$P(A\cup B)=P(A)+P(B)-P(A)P(B)$$

$$\Rightarrow \quad P(A\cup B).P(\overline{A}\cap\overline{B})\leq (P(A)+P(B)).(P(\overline{A}).P(\overline{B}))$$

$$\Rightarrow \quad P(A\cup B).P(\overline{A}\cap\overline{B})\leq P(A).P(\overline{A})P(\overline{B})+P(B).P(\overline{B}).P(\overline{A})$$

$$\Rightarrow \quad P(A\cup B).P(\overline{A}\cap\overline{B})\leq P(A)P(\overline{B})+P(B).P(\overline{A})$$

[since $(P(\overline{A}))$ and $(P(\overline{B}))$ are less than or equal to one]

$$\Rightarrow \quad P(A\cup B).P(\overline{A}\cap\overline{B})\leq P(C).$$

7. $y(x)=\int_{\pi^2/16}^{x^2}\frac{\cos x.\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}.d\theta$. Find $y'(x)$ at $x = \pi$.

[2]

Sol. $$y(x)=\int_{\pi^2/16}^{x^2}\frac{\cos x.\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$$

$$y'(x)=\cos x\left[\frac{d}{dx}\int_{\pi^2/16}^{x^2}\frac{\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta\right]-\sin x\int_{\pi^2/16}^{x^2}\frac{\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$$

$$=\frac{\cos x.\cos|x|}{1+(\sin|x|)^2}.2x-\sin x\int_{\pi^2/16}^{x^2}\frac{\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$$

$\Rightarrow$ $y'(\pi)=2\pi$

8. $p(x)=51x^{101}-2323x^{100}-45x+1035$

Using Rolle's theorem, prove that $p(x)$ has at least one root in $\left(45^{1/100},\ 46\right)$.

[2]

Sol. Let $F(x)=\frac{x^{102}}{2}-23x^{101}-\frac{45}{2}x^2+1035x+c$

As $F(x)$ is differentiable and continuous in $\left(45^{1/100},\ 46\right)$ and

$F\left(45^{1/100}\right)=F(46)=c$

By Rolle's theorem

$\therefore$ $F'(x)=p(x)=51x^{101}-2323x^{100}-45x+1035=0$

has at least one root in $\left(45^{1/100},\ 46\right)$.

9. If M is a 3×3 matrix, where det (M) = 1 and $MM^T = I$, then prove that det(M – I) = 0.

[2]

Sol. Let $M=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & t\end{bmatrix}$

Since $MM^T = I$

$\therefore$ M^T is inverse of M

$\Rightarrow$ M^T is adjoint M [Since det (M) = 1]

$$\Rightarrow \begin{bmatrix}a & d & g\\ b & e & h\\ c & f & t\end{bmatrix}=\begin{bmatrix}et-fh & ch-bt & bf-ce\\ gf-dt & at-cg & cd-af\\ dh-eg & bg-ah & ae-bd\end{bmatrix}\quad(1)$$

Now $|M-I|=\begin{vmatrix}a-1 & b & c\\ d & e-1 & f\\ g & h & t-1\end{vmatrix}$

On expanding above determinant and using (1)

$|M-I| = 0$

OR

For any matrix M there exist a real number λ and a non–zero column vector X such that

$(M-\lambda I)X = \mathbf{0}$

$\Rightarrow$ $MX = \lambda IX$

$\Rightarrow \quad MX = \lambda X \qquad (1)$

taking transpose of both side of (1)

$X^T M^T = \lambda X^T \qquad (2)$

Multiplying (1) and (2)

$X^T M^T MX = \lambda^2 X^T X$ as $(M^T M = I)$

$\Rightarrow \quad X^T IX = \lambda^2 X^T X$

$\Rightarrow \quad X^T X = \lambda^2 X^T X$

as X is a non zero column vector

$X^T X \neq 0$

$\Rightarrow \quad \lambda^2 = 1$

$\Rightarrow \quad \lambda = 1$ or -1

$\therefore$ when $\lambda = 1$

$(M - I)X = \mathbf{0}$

$\Rightarrow \quad |(M - I)X| = 0$

$\Rightarrow \quad |M - I| = 0$

10. Two lines having direction ratios (1, 0, –1) and (1, –1, 0) are parallel to a plane passing through (1, 1, 1). This plane cuts co–ordinate axes at A, B, C. Find volume of tetrahedron OABC.

[2]

Sol. Vector normal to the plane $= \left(\hat{i} + 0.\hat{j} - \hat{k}\right) \times \left(\hat{i} - \hat{j} + 0\hat{k}\right)$

$= -\left(\hat{i} + \hat{j} + \hat{k}\right)$

Direction ratios of normal to the plane is (1, 1, 1)

$\therefore$ Equation of plane: $x + y + z = 3$

$\Rightarrow \quad \overrightarrow{OA} = 3\hat{i}$

$\Rightarrow \quad \overrightarrow{OB} = 3\hat{j}$

$\Rightarrow \quad \overrightarrow{OC} = 3\hat{k}$

volume of tetrahedron $= \dfrac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \dfrac{9}{2}$ cubic unit

11. If *a*, *b*, *c* are positive numbers, then prove that $(1+a)^7(1+b)^7(1+c)^7 > 7^7 a^4 b^4 c^4$

[4]

Sol. A.M. ≥ G.M.

$$\frac{a+b+c+ab+bc+ca+abc}{7} \geq \left(a^4 b^4 c^4\right)^{1/7}$$

$$a+b+c+ab+bc+ca+abc \geq 7\left(a^4 b^4 c^4\right)^{1/7}$$

$$\Rightarrow \quad 1+a+b+c+ab+bc+ca+abc > 7\left(a^4 b^4 c^4\right)^{1/7}$$

$$\Rightarrow \quad (1+a)(1+b)(1+c) > 7\left(a^4 b^4 c^4\right)^{1/7}$$

$$\Rightarrow \quad (1+a)^7(1+b)^7(1+c)^7 > 7^7\left(a^4 b^4 c^4\right)$$

12. A curve passes through (2, 0) and slope at point P(*x*, *y*) is $\dfrac{(x+1)^2 + (y-3)}{(x+1)}$. Find equation of curve and area between curve and x–axis in 4th quadrant.

[4]

Sol. $\frac{dy}{dx} = \frac{(x+1)^2 + (y-3)}{(x+1)}$

$(x+1)\frac{dy}{dx} = (x+1)^2 + (y-3)$

Let $X = x+1, Y = y-3$

$$X\frac{dY}{dX} = X^2 + Y$$

$$\frac{dY}{dX} - \frac{Y}{X} = X$$

I.F = $e^{\int -\frac{1}{X}.dX} = e^{-\log X} = \frac{1}{X}$

Solution is

$$Y.\frac{1}{X}. = \int \frac{1}{X}.X.dX$$

$$\frac{Y}{X} = X + C$$

$$\frac{y-3}{x+1} = x+1+C \qquad (1)$$

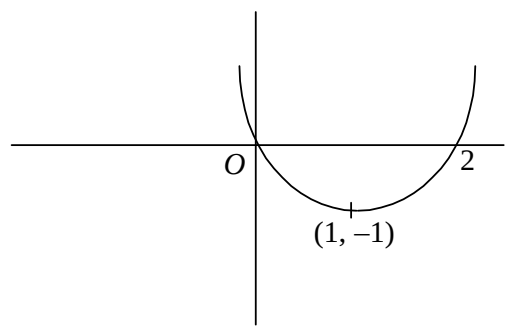
As curve passes through (2, 0) C = –4

$\therefore$ (1) becomes

$$(x-3)(x+1) = (y-3)$$

Area $= \int_0^2 (x^2 - 2x)dx = \left[\frac{x^3}{3} - x^2\right]_0^2$

$= \left|\frac{8}{3} - 4\right| = \frac{4}{3}$



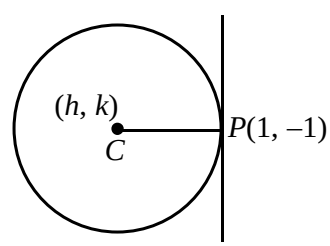
13. Line $2x + 3y + 1 = 0$ is a tangent to a circle at (1, –1). This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points (0, –1) and (–2, 3). Find equation of circle.

[4]

Sol. Slope CP is $\frac{k+1}{h-1} = \frac{3}{2}$

$$2k + 2 = 3h - 3$$

$$3h - 2k = 5 \qquad (1)$$



Equation of second circle is

$$(x-0)(x+2) + (y+1)(y-3) = 0$$

$$x^2 + y^2 + 2x - 2y - 3 = 0 \qquad (2)$$

Centre (–1, 1) and radius = $\sqrt{5}$

Above circle is orthogonal to the circle having centre (h, k).

$\therefore \quad (h-1)^2 + (k+1)^2 + 5 = (h+1)^2 + (k-1)^2$

$$4h - 4k = 5 \qquad (3)$$

Solving (1) and (3)

$$h = \frac{5}{2},\ k = \frac{5}{4}$$

$\therefore$ Equation of circle is

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{5}{4}\right)^2=\frac{117}{16}.$$

14. $$f(x)=\begin{cases} b\sin^{-1}\left(\dfrac{x+c}{2}\right), & -\dfrac{1}{2}<x<0 \\ \dfrac{1}{2} & at\ \ x=0 \\ \dfrac{e^{ax/2}-1}{x}, & 0<x<\dfrac{1}{2} \end{cases}.$$

If $f(x)$ is differentiable at $x = 0$. Find the value of a also prove that $64b^2 = 4 - c^2$.

[4]

Sol. RHD $=\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{\dfrac{e^{ah/2}-1}{h}-\dfrac{1}{2}}{h}$

$$\lim_{h\to 0}\frac{1+\dfrac{ah}{2}+\dfrac{a^2h^2}{2^2}.\dfrac{1}{2!}.....-1-\dfrac{h}{2}}{h^2}$$

For the limit to exist $a = 1$;

Hence RHD $=\dfrac{1}{8}$

LHD $=\lim\limits_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}$

$$\lim_{h\to 0}\frac{b\sin^{-1}\left(\dfrac{c-h}{2}\right)-\dfrac{1}{2}}{-h}$$

$$\lim_{h\to 0}\frac{b\left\{\dfrac{c-h}{2}+\dfrac{1^2}{3!}\left(\dfrac{c-h}{2}\right)^3+\dfrac{1^2.3^2}{5!}\left(\dfrac{c-h}{2}\right)^5.....\right\}-\dfrac{1}{2}}{-h}$$

As function is differentiable so this limit is equal to $\dfrac{1}{8}$

For this constant part must be zero and coefficient of h in the numerator must be equal to $-\dfrac{1}{8}$

Coefficient of h in numerator is equal to

$$-\frac{b}{2}\left[1+\frac{1^2}{3!}3\left(\frac{c}{2}\right)^2+\frac{1^2.3^2}{5!}\times 5\left(\frac{c}{2}\right)^4+....\right]=-\frac{1}{8} \qquad (1)$$

Clearly left hand side is derivative of

$-b\sin^{-1}\dfrac{x}{2}$ at x = c

$\Rightarrow$ Left hand side of equation (1) is

$$-b\frac{1}{\sqrt{1-\frac{c^2}{4}}}.\frac{1}{2}=-\frac{1}{8}$$

$\Rightarrow$ Squaring both side

$$64b^2=4-c^2$$

15. A box contains 6 white and 12 red balls. 6 balls are drawn without replacement, in which at least 4 balls are white, find the probability that exactly one of the ball in next two draws, is white.

[4]

Sol. Let E be the event when there are minimum 4 white balls in Ist 6 draws (without replacement)

$$E=E_4\cup E_5\cup E_6$$

$E_4\equiv$ exactly 4 white balls are drawn

$E_5\equiv$ exactly 5 white balls are drawn

$E_6\equiv$ exactly 6 white balls are drawn

B be the event such that out of next two drawn exactly one is white.

$$P(B)=P(B\cap E_4)+P(B\cap E_5)+P(B\cap E_6)$$

$$=P(B/E_4).P(E_4)+P(B/E_5).P(E_5)+P(B/E_6).P(E_6)$$

$$=\frac{{}^2C_1.{}^{10}C_1}{{}^{12}C_2}\frac{{}^6C_4.{}^{12}C_2}{{}^{18}C_6}+\frac{{}^1C_1.{}^{11}C_1}{{}^{12}C_2}\frac{{}^6C_5.{}^{12}C_1}{{}^{18}C_6}+0$$

16. $y^2-2y-4x+5=0$ is an equation of parabola. A point P is taken on the parabola and tangent at P intersects directrix at Q. Find the locus of a point R such that it divides PQ externally in the ratio $\frac{1}{2}:1$.

[4]

Sol. Parabola given $(y-1)^2=4(x-1)$

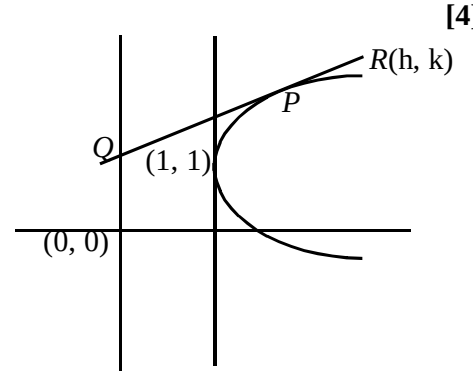
Equation to directrix $x=0$ focus (2, 1)

Let point $P(t^2+1,2t+1)$ is on the parabola

Tangent at any point P

$$ty-x=t^2+t-1$$

Co-ordinate of point Q$\left(0,\ \frac{t^2+t-1}{t}\right)$



Let the point $R(h, k)$ whose locus is to be found for which R divides PQ externally in the ratio 1 : 2

$$h=\frac{2(t^2+1)-1.0}{2-1}=2t^2+2\Rightarrow t^2=\frac{h-2}{2}\qquad(1)$$

$$k=\frac{2(2t+1)-1.\left(\frac{t^2+t-1}{t}\right)}{2-1}=\frac{3t+\frac{1}{t}+1}{1}=\frac{3t^2+t+1}{t}\qquad(2)$$

Using (1) and (2)

$$\Rightarrow\quad k=\frac{3(t^2+1)+(t-2)}{t}$$

$$\Rightarrow \qquad k = \frac{\frac{3h}{2} - 2}{t} + 1$$

$$\Rightarrow \qquad (k-1)^2 = \frac{(3h-4)^2}{4t^2} \qquad \Rightarrow \qquad (k-1)^2 = \frac{(3h-4)^2}{2(h-2)}$$

The locus of point R is

$$2(y-1)^2(x-2) = (3x-4)^2$$

17. Prove that $\sin x + 2x \geq \frac{3x(x+1)}{\pi}$, $x \in \left[0, \frac{\pi}{2}\right]$. Justify the inequalities used in the relation.

[4]

Sol. Let $f(x) = \sin x + 2x$

$\therefore \qquad f'(x) = \cos x + 2$

$f''(x) = -\sin x$

For $x \in \left(0, \frac{\pi}{2}\right) \qquad f'(x) > 0, \ f''(x) < 0$

$\Rightarrow \qquad f(x)$ is increasing function and concave downward

$$g(x) = \frac{3x(x+1)}{\pi}$$

$$g'(x) = \frac{6x+3}{\pi}$$

$$g''(x) = \frac{6}{\pi}$$

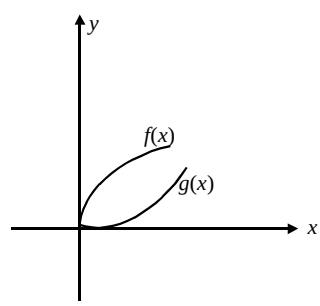
For $x \in \left(0, \frac{\pi}{2}\right) \quad g'(x) > 0, \ g''(x) > 0$

$g(x)$ is increasing function and concave upward

Also $f\left(\frac{\pi}{2}\right) > g\left(\frac{\pi}{2}\right)$

$f(0) = g(0) = 0$

Hence from graph easily see $f(x) \geq g(x)$



18. $\displaystyle\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx$.

[4]

Sol. $$\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx = \int_{-\pi/3}^{\pi/3} \frac{\pi}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} + \int_{-\pi/3}^{\pi/3} \frac{4x^3 dx}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} = I_1 + I_2$$

$I_2 = 0$ [odd function]

$$I_1 = 2\pi \int_0^{\pi/3} \frac{\sec^2\left(\frac{x}{2} + \frac{\pi}{6}\right)}{1 + 3\tan^2\left(\frac{x}{2} + \frac{\pi}{6}\right)} dx$$

$$= \frac{4\pi}{3}\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{dt}{\frac{1}{3}+t^2} = \frac{4\pi}{3}\times\sqrt{3}\left[\tan^{-1} t\sqrt{3}\right]_{1/\sqrt{3}}^{\sqrt{3}} = \frac{4\pi}{\sqrt{3}}\left[\tan^{-1} 3 - \frac{\pi}{4}\right]$$

19. $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$

$U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$, $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$.

If AX = U has infinitely many solution. Prove that BX = V has no unique solution, also prove that if afd $\neq$ 0, then BX = V has no solution. X is a vector.

[4]

Sol. AX = U

$$\Rightarrow \quad \begin{bmatrix} a & 1 & 0 & : & x \\ 1 & b & d & : & y \\ 1 & b & c & : & z \end{bmatrix} = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 1-ab & 0-ad & : & x \\ 1 & b & d & : & y \\ 0 & 0 & d-c & : & z \end{bmatrix} = \begin{bmatrix} f-ag \\ g \\ h-g \end{bmatrix}$$

as AX = U has infinitely many solution

$\Rightarrow$ $d = c$ and $h = g$

Now BX = V

$$\Rightarrow \quad \begin{bmatrix} a & 1 & 1 & : & x \\ 0 & d & c & : & y \\ f & g & h & : & z \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a & 1 & 1 & : & x \\ 0 & d & c & : & y \\ 0 & 0 & h-\frac{f}{a}-\frac{c}{d}\left(g-\frac{f}{a}\right) & : & z \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ -af \end{bmatrix} \qquad (1)$$

If c = d = 0 and h = g

clearly BX = V has no unique solution

if c = d $\neq$ 0 $h-\frac{f}{a}-\frac{c}{d}\left(g-\frac{f}{a}\right)=0$

$\Rightarrow$ $[A : X] = 0$

$\Rightarrow$ $BX = V$ has no unique solution

Now as afd $\neq$ 0

$\Rightarrow$ $a \neq 0$, f $\neq$ 0 d $\neq$ 0

$\Rightarrow$ [A : X] = 0 but as –af $\neq$ 0

$\therefore$ from equation (1)

BX = V has no solution.

20. P_1 and P_2 are planes passing through origin. L_1 and L_2 also passes through origin. L_1 lies on P_1 not on P_2 and L_2 lies on P_2 but not on P_1. Show that there exists points A, B, C and whose permutation $A'.B'.C'$ can be chosen such that

(i) A is on L_1, B on P_1 but not on L_1 and C not on P_1.

(ii) A' in on L_2, B' on P_2 but not on L_2 and C' not on P_2.

[4]

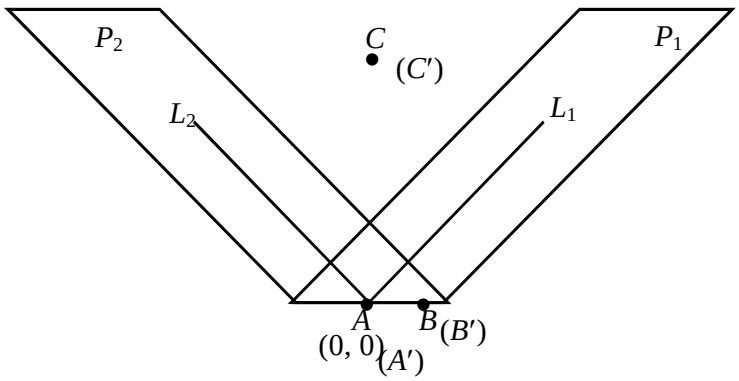
Sol. We take

$A = A' =$ origin

$B = B'$ = any point other than origin on the line of intersection of P_1 and P_2.

$C = C'$ = any point neither on P_1 nor on P_2

In this case both conditions (i) and (ii) are fulfilled.

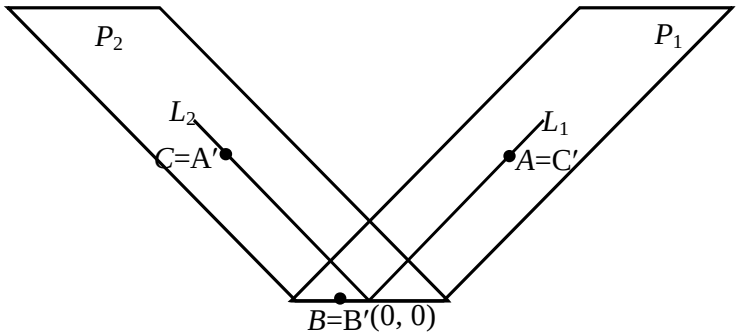


Similarly if we take

A = non–origin point on L_1

B = non–origin point on the line of intersection of P_1 and P_2

C = non–origin point on L_2

If we take A = C′, B = B′ and C = A′

Both the conditions (i) and (ii) are fulfilled