SOLUTIONS TO IIT–JEE 2005 (SCREENING)
MEMORY BASED QUESTIONS
MATHEMATICS

1. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P., where $\Delta = b^2 - 4ac$, then

(a) $\Delta \neq 0$ (b) $b\Delta = 0$

(c) $cb \neq 0$ (d) $c\Delta = 0$

Sol. **d**

$$\left(\alpha^2 + \beta^2\right)^2 = (\alpha + \beta)\left(\alpha^3 + \beta^3\right)$$

$$\left(\frac{b^2 - 2ac}{a^2}\right)^2 = \left(-\frac{b}{a}\right)\left(\frac{-b^3 + 3abc}{a^3}\right)$$

$\Rightarrow \quad 4a^2c^2 = acb^2$

$\Rightarrow \quad ac\left(b^2 - 4ac\right) = 0$

As $a \neq 0 \Rightarrow c\Delta = 0$.

2. Minimum area of the triangle formed by any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the coordinate axis is

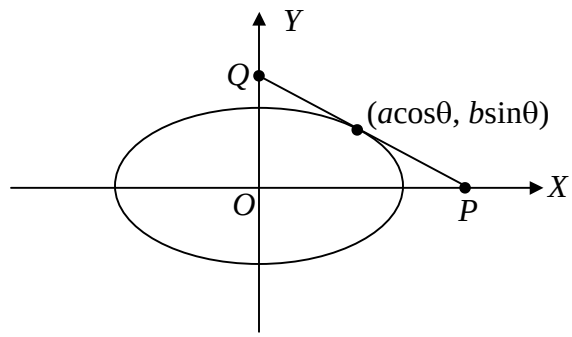
(a) $\frac{a^2 + b^2}{2}$ (b) $\frac{(a+b)^2}{2}$

(c) ab (d) $\frac{(a-b)^2}{2}$

Sol. **c**

Equation of tangent at ($a\cos\theta$, $b\sin\theta$) is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$



$$P \equiv \left(\frac{a}{\cos\theta}, 0\right) \quad Q \equiv \left(0, \frac{b}{\sin\theta}\right)$$

Area of OPQ =

$$\frac{1}{2}\cdot\left|\left(\frac{a}{\cos\theta}\right)\cdot\left(\frac{b}{\sin\theta}\right)\right|$$

$$= \frac{ab}{|\sin 2\theta|}$$

$(\text{Area})_{\min} = ab$.

3. The integral value of $\int_{-2}^{0}\left[x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right]dx$ is

(a) 2 (b) 4

(c) 0 (d) 8

Sol. **b**

Let $x + 1 = t$

$$I = \int_{-1}^{1}\left(t^3 + 2 + t\cos t\right)dt$$

$$= \int_{-1}^{1} 2dt = 2 \times 2 = 4.$$

4. If $f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ x, & \text{when } x \text{ is irrational} \end{cases}$, $g(x) = \begin{cases} x, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$

then $f - g$ is

(a) one–one onto (b) one–one not onto

(c) not one–one but onto (d) not one–one not onto

Sol. **a**

$$(f-g)(x) = \begin{cases} -x & x \in Q \\ x & x \notin Q \end{cases}$$

it is obvious that function is one–one onto.

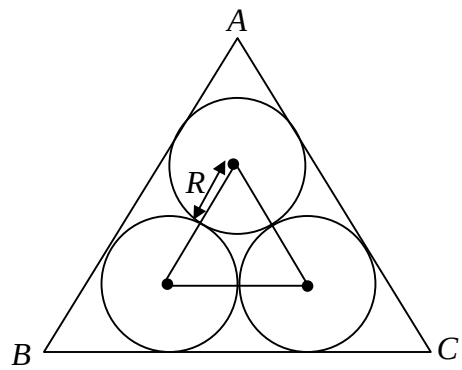
5. The area of the equilateral triangle which containing three coins of unity radius is

(a) $6+4\sqrt{3}$ sq. units

(b) $8+\sqrt{3}$ sq. units

(c) $4+\frac{7\sqrt{3}}{2}$ sq. units

(d) $12+2\sqrt{3}$ sq. units



Sol. **a**

In ΔBC_1M,

$BM = (C_1M).\cot 30^o$

$\Rightarrow$ $BM = \sqrt{3}$

Similarly, $CN = \sqrt{3}$

and, $MN = C_1C_2 = 1 + 1 = 2$

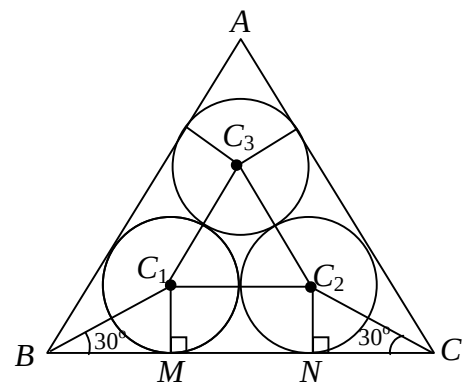
Hence, side BC

$= \sqrt{3}+\sqrt{3}+2 = 2\left(1+\sqrt{3}\right)$

$\Rightarrow$ Area of equilateral triangle

$= \frac{\sqrt{3}}{4}\left[2\left(1+\sqrt{3}\right)\right]^2$

$= 6+4\sqrt{3}$ sq. units.



6. A plane is at unit distance from origin. It cut coordinate axis at P, Q, R respectively. If the locus of centroid of the ΔPQR is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then $k =$

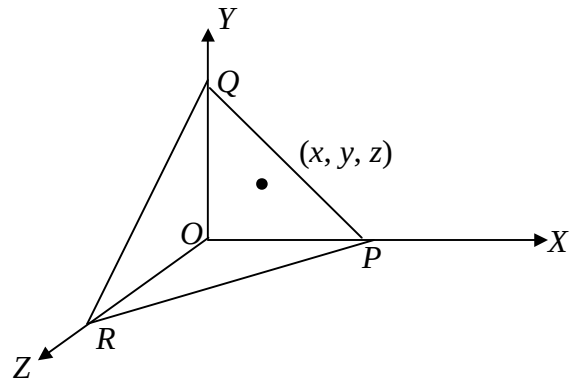
(a) 3 (b) 1

(c) 2 (d) 9

Sol. **d**

Let equation of plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad (1)$$



$\Rightarrow$ $P \equiv (a, 0, 0)$,

$Q \equiv (0, b, 0)$, $R \equiv (0, 0, c)$

Since, distance of this plane from origin is unity

$$\Rightarrow \quad \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1$$

$$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1 \qquad (2)$$

Now, centroid of plane (1) is (x, y, z)

$$\Rightarrow \quad x=\frac{a}{3},\ y=\frac{b}{3} \text{ and } z=\frac{c}{3} \qquad (3)$$

So, $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{9}{a^2}+\frac{9}{b^2}+\frac{9}{c^2}$

$$\Rightarrow \quad k=9\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=9 \qquad \text{[from (2)]}$$

$\Rightarrow$ $k = 9$

7. If ω is a cube root of unity but not equal to 1 then minimum value of $|a + b\omega + c\omega^2|$ (where a, b, c are integers but not all equal) is

(a) 0 (b) $\frac{\sqrt{3}}{2}$

(c) 1 (d) 2

Sol. **c**

Let $y = |a+b\omega+c\omega^2|$

For y to be minimum y^2 must be minimum

$$y^2 = |a+b\omega+c\omega^2|^2$$

$$= (a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$$

$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

Since a, b and c are not all equal at a time

So, minimum value of y^2 occurs when any two are same and third is differ by 1.

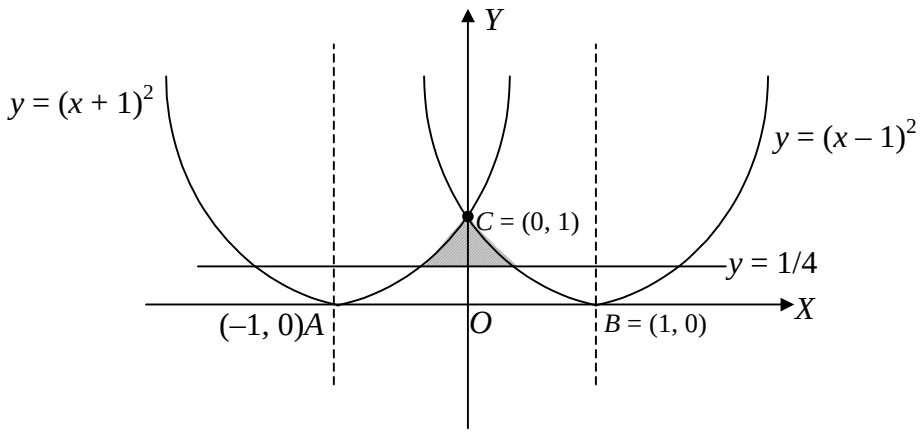
$\Rightarrow$ minimum of $y = 1$ (as a, b, c are integers)

8. The area bounded by the curve $y=(x+1)^2, y=(x-1)^2$ and the line $y=\frac{1}{4}$ is

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

Sol. **d**



Required area = $2\left|\int_{1/4}^{1}\left(\sqrt{y}-1\right)dy\right|$ [from the symmetry]

$$= 2\left|\left[\frac{y^{3/2}}{\frac{3}{2}}-y\right]_{1/4}^{1}\right|$$

$$= 2\left|\left[\left(\frac{2}{3}-1\right)-\left(\frac{2}{3}.\frac{1}{8}-\frac{1}{4}\right)\right]\right|$$

$$= 2\left|\left[-\frac{1}{3}+\frac{1}{4}-\frac{1}{12}\right]\right|$$

$= \frac{1}{3}$ sq. unit.

9. $f(x)=||x|-1|$ is not differentiable at

(a) 0 (b) ± 1, 0

(c) 1 (d) ± 1

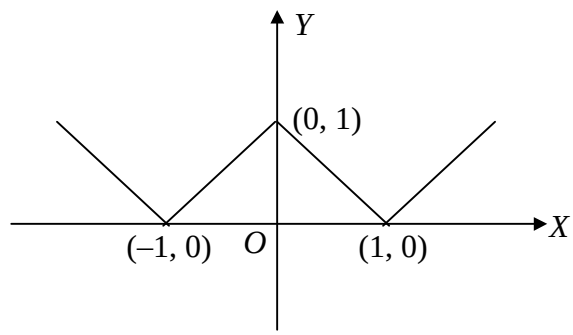
Sol. **b**

$f(x) = ||x|-1|$

$= \begin{cases} |x|-1, & |x|-1 \geq 0 \\ -|x|+1, & |x|-1 < 0 \end{cases}$

$= \begin{cases} |x|-1, & x \leq -1 \text{ or } x \geq 1 \\ -|x|+1, & -1 < x < 1 \end{cases}$

$= \begin{cases} -x-1, & x \leq -1 \\ x+1, & -1 < x < 0 \\ -x+1, & 0 \leq x < 1 \\ x-1, & x \geq 1 \end{cases}$



From the graph, it is clear that $f(x)$ is not differentiable at $x = -1$, 0 and 1.

10. Tangent to the parabola $y = x^2 + 6$ at (1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at the point

(a) (–6, –9) (b) (–13, –9)
(c) (–6, –7) (d) (13, 7)

Sol. **c**

Equation of tangent at (1, 7) to $y = x^2 + 6$ is given by

$$\frac{1}{2}(y+7) = x.1 + 6$$

$\Rightarrow \quad y = 2x + 5 \qquad (1)$

This tangent also touches the circle

$$x^2 + y^2 + 16x + 12y + c = 0 \qquad (2)$$

Now solving (1) and (2), we get

$\Rightarrow \quad x^2 + (2x+5)^2 + 16x + 12(2x+5) + c = 0$

$\Rightarrow \quad 5x^2 + 60x + 85 + c = 0$

Since, roots are equal

$\Rightarrow \quad x = -\frac{60}{10} = -6$

$\Rightarrow \quad y = -7$

Hence, point of contact is (–6, –7).

11. The locus of centre of the circle which touches the circle $x^2 + (y-1)^2 = 1$ externally and also touches x–axis is

(a) $\{(x, y): x^2 + (y-1)^2 = 4\} \cup \{(x, y): y < 0\}$

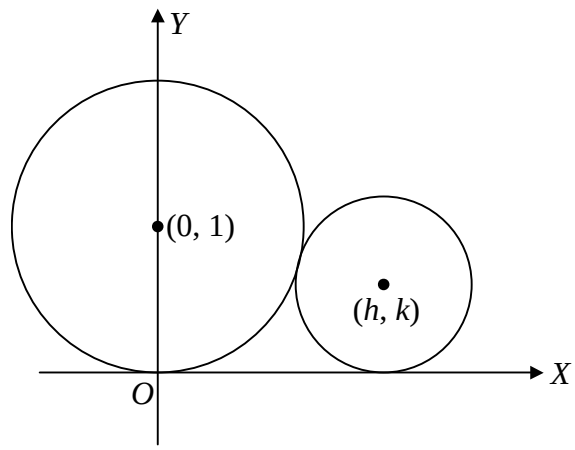
(b) $\{(x, y): x^2 = 4y\} \cup \{(0, y): y < 0\}$

(c) $\{(x, y): x^2 = y\} \cup \{(0, y): y < 0\}$

(d) $\{(x, y): x^2 = 4y\} \cup \{(x, y): y > 0\}$

Sol. **b**

According to given condition



$$\sqrt{(h-0)^2 + (k-1)^2} = 1 + |k|$$

$$\Rightarrow \quad h^2 + (k-1)^2 = (1 + |k|)^2$$

$$\Rightarrow \quad h^2 = 2k + 2|k|$$

Hence locus is $x^2 = 2y + 2|y|$

$y > 0 \quad x^2 = 4y$

$y < 0 \quad x^2 = 0 \quad \Rightarrow x = 0.$

12. In a throw of a dice the probability of getting one in even number of throw is

(a) $\frac{5}{36}$ (b) $\frac{5}{11}$

(c) $\frac{6}{11}$ (d) $\frac{1}{6}$

Sol. **b**

Required probability

$$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \ldots$$

$$= \frac{\frac{5}{6} \cdot \frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{\frac{5}{36}}{\frac{11}{36}} = \frac{5}{11}.$$

13. If $\int_{\sin x}^{1} t^2 f(t)\,dt = 1 - \sin x$, $x \in \left(0, \frac{\pi}{2}\right)$ then $f\left(\frac{1}{\sqrt{3}}\right)$ equal to

(a) 3 (b) $\frac{1}{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

Sol. **a**

On differentiating both sides, we get

$\Rightarrow \quad -\sin^2 x \; f(\sin x)\cos x = -\cos x$

$\Rightarrow \quad f(\sin x) = \cos ec^2 x$

$\Rightarrow \quad f(x) = \dfrac{1}{x^2}$

$\Rightarrow \quad f\left(\dfrac{1}{\sqrt{3}}\right) = 3$

14. $f(x)$ is twice differentiable polynomial function such that $f(1) = 1,\ f(2) = 4,\ f(3) = 9$, then

(a) $f''(x) = 2,\ \forall x \in R$

(b) There exist at least one $x \in (1\ 3)$ such that $f''(x) = 2$

(c) There exist at least one $x \in (2, 3)$ such that $f'(x) = 5 = f''(x)$

(d) There exist at least one $x \in (1, 2)$ such that $f''(x) = 3$

Sol. **b**

Let a function be $g(x) = f(x) - x^2$

$\Rightarrow$ $g(x)$ has at least 3 real roots which are $x = 1, 2, 3$.

$\Rightarrow$ $g'(x)$ has at least 2 real roots in $x \in (1, 3)$

$\Rightarrow$ $g''(x)$ has at least 1 real root in $x \in (1, 3)$

$\Rightarrow$ $f''(x) = 2$ for at least one $x \in (1, 3)$

15. The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} + \ldots\ldots + \binom{30}{20}\binom{30}{30}$

(a) ${}^{60}C_{20}$ (b) ${}^{30}C_{10}$

(c) ${}^{60}C_{30}$ (d) ${}^{40}C_{30}$

Sol. **b**

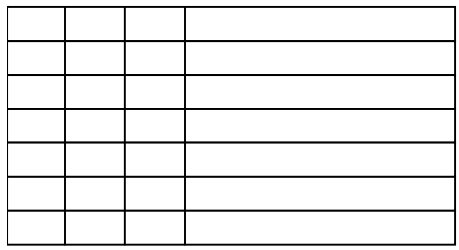
$$(1-x)^{30} = {}^{30}C_0 x^0 - {}^{30}C_1 x^1 + {}^{30}C_2 x^2 + \ldots\ldots + (-1)^{30}\ {}^{30}C_{30} x^{30} \quad (1)$$

$$(x+1)^{30} = {}^{30}C_0 x^{30} + {}^{30}C_1 x^{29} + {}^{30}C_2 x^{28} + \ldots. + {}^{30}C_{10}\ x^{20} + \ldots.. + {}^{30}C_{30} x^0 \quad (2)$$

Multiplying (1) and (2) and equating the coefficient of x^{20} on both sides, we get

Required sum = coefficient of x^{20} in $(1 - x^2)^{30} = {}^{30}C_{10}$

16. There is a rectangular sheet of dimension $(2m - 1) \times (2n - 1)$, (where $m > 0$, $n > 0$). It has been divided into square of unit area by drawing lines perpendicular to the sides. Find number of rectangles having sides of odd unit length.



(a) $(m+n+1)^2$

(b) $mn(m+1)(n+1)$

(c) 4^{m+n-2}

(d) m^2n^2

Sol. **d**

Along horizontal side one unit can be taken in $(2m - 1)$ ways and 3 unit side can be taken in $2m - 3$ ways

$\therefore$ The number of ways of selecting a side horizontally is

$(2m-1+2m-3+2m-5+.....+3+1)$

Similar the number of ways along vertical side is

$(2n-1+2n-3+.....+5+3+1)$

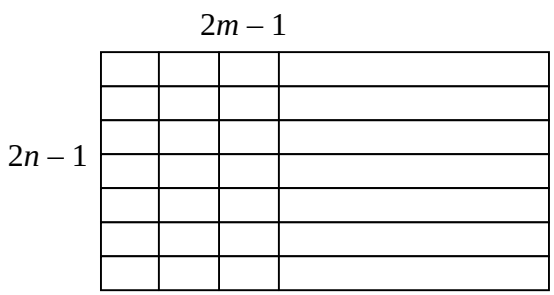
$\therefore$ total number of rectangles

$= (1 + 3 + 5 + \ldots + 2m - 1)$

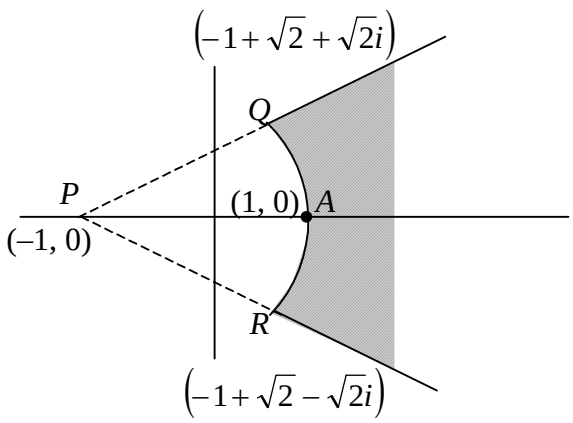
$\times (1 + 3 + 5 + \ldots + 2n - 1)$

$= \dfrac{m(1+2m-1)}{2} \times \dfrac{n(1+2n-1)}{2}$

$= m^2n^2.$



17. PQ and PR are two infinite rays, QAR is an arc. Point lying in the shaded region excluding the boundary satisfies



(a) $|z-1| > 2 : |\arg(z-1)| < \dfrac{\pi}{4}$

(b) $|z-1| > 2 : |\arg(z-1)| < \dfrac{\pi}{2}$

(c) $|z+1| > 2 : |\arg(z+1)| < \dfrac{\pi}{4}$

(d) $|z+1| > 2 : |\arg(z+1)| < \dfrac{\pi}{2}$

Sol. **c**

Equation of ray PQ $\arg(z + 1) = \frac{\pi}{4}$

Equation of ray PR $\arg(z + 1) = -\frac{\pi}{4}$

$\therefore$ shaded region is $-\frac{\pi}{4} < \arg(z + 1) < \frac{\pi}{4}$

$|\arg(z+1)| < \frac{\pi}{4}$

$|PQ| = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = 2$

$|PA| = 2$

$|PR| = 2$

So arc QAR is of a circle of radius 2 unit with centre at $P(-1, 0)$

$\Rightarrow$ all the points in the shaded region are exterior to this circle $|z + 1| = 2$

$\Rightarrow$ $|z+1| > 2$ and $|\arg(z + 1)| < \frac{\pi}{4}$

18. $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{vmatrix}, \quad I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$A^{-1} = \frac{1}{6}\left(A^2 + CA + DI\right)$ then C and D equal to

(a) –11, 6 (b) –6, 11

(c) 6, 11 (d) –6, –11

Sol. **b**

Here

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}, \quad A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$\Rightarrow$ $$\frac{1}{6}\left(A^2 + CA + DI\right) = \frac{1}{6}\begin{bmatrix} 1+C+D & 0 & 0 \\ 0 & -1+C+D & 5+C \\ 0 & -10-2C & 14+4C+D \end{bmatrix}$$

$\Rightarrow$ $$\frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 1+C+D & 0 & 0 \\ 0 & -1+C+D & 5+C \\ 0 & -10-2C & 14+4C+D \end{bmatrix}$$

$\Rightarrow$ $1 + C + D = 6$ (1)

and $5 + C = -1$

$\Rightarrow$ $C = -6$

and $D = 11$.

19. $(x^2 + y^2)dy = xydx$. If $y(x_0) = e$, $y(1) = 1$, then value of $x_0 =$

(a) $\sqrt{3}e$ (b) $\sqrt{e^2 - \frac{1}{2}}$

(c) $\sqrt{\frac{e^2 - 1}{2}}$ (d) $\sqrt{\frac{e^2 + 1}{2}}$

Sol. **a**

$\Rightarrow \quad x^2dy + y^2dy = xydx$

$\Rightarrow \quad x(xdy - ydx) = -y^2dy$

$\Rightarrow \quad x\frac{(ydx - xdy)}{y^2} = dy$

$\Rightarrow \quad \frac{x}{y}d\left(\frac{x}{y}\right) = \frac{dy}{y}$

Integrating

$\Rightarrow \quad \frac{x^2}{2y^2} = \log_e y + c$

given $y(1) = 1 \Rightarrow c = \frac{1}{2}$

$\Rightarrow \quad \frac{x^2}{2y^2} = \log_e y + \frac{1}{2}$

Now $y(x_0) = e$

$\Rightarrow \quad \frac{x_0^2}{2e^2} - \log_e e - \frac{1}{2} = 0$

$\Rightarrow \quad x_0^2 = 3e^2$

$\Rightarrow \quad x_0 = \sqrt{3}e$.

20. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and $Q = PAP^T$, then $P(Q^{2005})P^T$ equal to

(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \sqrt{3}/2 & 2005 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2005 \\ \sqrt{3}/2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & \sqrt{3}/2 \\ 0 & 2005 \end{bmatrix}$

Sol. **a**

If $Q = PAP^T$

$P^T Q = AP^T$ (1) (as $PP^T = I$)

$P^T Q^{2005} P = AP^T Q^{2004} P$

$= A^2 P^T Q^{2003} P$

$= A^3 P^T Q^{2002} P$

$= A^{2004} P^T (QP)$

$= A^{2004} P^T (PA)$ ($Q = PAP^T \Rightarrow QP = PA$)

$= A^{2005}$

$\Rightarrow$ $A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

21. If $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, $-\pi < \alpha, \beta < \pi$, then total number of ordered pair of (α, β) is

(a) 0 (b) 1

(c) 2 (d) 4

Sol. **d**

$-2\pi < \alpha - \beta < 2\pi$

$\cos(\alpha - \beta) = 1$

$\Rightarrow$ $\alpha - \beta = 0$

$\Rightarrow$ $\alpha = \beta$

$\cos 2\alpha = \frac{1}{e}$ and $-2\pi < 2\alpha < 2\pi$

$\Rightarrow$ There will be four solutions.

22. If $xdy = y(dx + ydy)$, $y > 0$ and $y(1) = 1$, then $y(-3)$ is equal to

(a) 1 (b) 3

(c) 5 (d) –1

Sol. **b**

$\Rightarrow$ $xdy - ydx = y^2\, dy$

$\Rightarrow$ $\frac{xdy - ydx}{y^2} = dy$

$\Rightarrow \quad -d\left(\frac{x}{y}\right) = dy$

$\Rightarrow \quad \frac{x}{y} + y = c$

$\because \quad y(1) = 1 \Rightarrow c = 2$

$\Rightarrow \quad \frac{x}{y} + y = 2$

for $x = -3$

$\Rightarrow \quad y^2 - 2y - 3 = 0 \quad \Rightarrow y = -1$ or 3

$\Rightarrow \quad y = 3 \quad (\because y > 0)$

23. If $f(x)$ is a differentiable function such that $f : R \to R$ and $f\left(\frac{1}{n}\right) = 0$ $\forall\ n \geq 1, n \in I$, then

(a) $f(x) = 0\ \forall\ x \in (0, 1]$ (b) $f(0) = 0 = f'(0)$

(c) $f(0) = 0$ but $f'(0)$ may or may not be 0 (d) $|f(x)| \leq 1;\ \forall x \in [0, 1]$

Sol. **b**

$$f(1) = f\left(\frac{1}{2}\right) = f\left(\frac{1}{3}\right) = ... = \lim_{n\to\infty} f\left(\frac{1}{n}\right) = 0$$

Since there are infinitely many points in $x \in (0, 1]$ where $f(x) = 0$

and $\lim_{n\to\infty} f\left(\frac{1}{n}\right) = 0$

$\Rightarrow \quad f(0) = 0.$

And since there are infinitely many points in the neighbourhood of $x = 0$ such that $f(x) = 0$

$\Rightarrow \quad f(x)$ remains constant in the neighbourhood of $x = 0$.

$\Rightarrow \quad f'(0) = 0$.

24. Which of the following is true in a triangle ABC

(a) $(b+c)\sin\frac{B-C}{2} = 2a\cos\frac{A}{2}$

(b) $(b+c)\cos\frac{A}{2} = 2a\sin\frac{B-C}{2}$

(c) $(b-c)\cos\frac{A}{2} = a\sin\frac{B-C}{2}$

(d) $(b-c)\sin\frac{B-C}{2} = 2a\cos\frac{A}{2}$

Sol. **c**

$$\frac{b-c}{a} = \frac{\sin B - \sin C}{\sin A} = \frac{2\sin\frac{B-C}{2}\cos\frac{B+C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$= \frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}}$$

$$\Rightarrow \quad (b-c)\cos\frac{A}{2} = a\sin\frac{B-C}{2}$$

25. If X and Y are two non–empty sets where $f : X \to Y$ is function is defined such that $f(c) = \{f(x) : x \in C\}$ for $C \subseteq X$ and $f^{-1}(D) = \{x : f(x) \in D)\}$ for $D \subseteq Y$, for any $A \subseteq X$ and $B \subseteq Y$, then

(a) $f^{-1}(f(A)) = A$

(b) $f^{-1}(f(A)) = A$ only if $f(x) = Y$

(c) $f(f^{-1}(B)) = B$ only if $B \subseteq f(X)$

(d) $f(f^{-1}(B)) = B$

Sol. **c**

The set B satisfied the above definition of function f so option (c) is correct.

26. If S is a set of $P(x)$ and $P(x)$ is polynomial of degree ≤ 2 such that $P(0) = 0,\ P(1) = 1,\ P'(x) > 0 \quad \forall x \in (0, 1)$, then

(a) $S = \phi$

(b) $S = ax + (1-a)x^2 \ \ \forall a \in (0, \infty)$

(c) $S = ax + (1-a)x^2 \ \ \forall a \in R$

(d) $S = ax + (1-a)x^2 \ \ \forall a \in (0, 2)$

Sol. **d**

Let $P(x) = bx^2 + ax + c$

As $P(0) = 0 \ \Rightarrow \ c = 0$

As $P(1) = 1 \ \Rightarrow a + b = 1$

$\therefore \quad P(x) = ax + (1-a)x^2$

Now $P'(x) = a + 2(1-a)x$

As $P'(x) > 0$ for $x \in (0, 1)$

$\Rightarrow$ only option (d) satisfies above condition.

27. If $y\cos x + x\cos y = \pi$, then $y''(0)$ is

(a) 1 (b) π

(c) 0 (d) $-\pi$

Sol. **b**

$$y\cos x + x\cos y = \pi$$

Differentiate both sides with respect to x, we get

$$-y\sin x + \cos x.y' + x(-\sin y)y' + \cos y = 0$$

Again differentiate with respect to x,

$$-y''\sin x - y\cos x + \cos x.y'' - y'\sin x - y'\sin y - x\left[\cos y\ y'^2 + \sin y\ y''\right] - y'\sin y = 0$$

Putting $x = 0$, we get

$$-y + y'' - 2\sin y\ y' = 0$$

$$y'' = y + 2y'\sin y$$

Since at $x = 0$, $y = \pi$

$$(y'')_0 = \pi.$$

28. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non–zero, non–coplanar vectors and $\vec{b}_1 = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2 = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$ and $\vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{c}.\vec{b}}{|\vec{b}|^2}\vec{b}$, $\vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{c}.\vec{b}_1}{|\vec{b}_1|^2}\vec{b}_1$, $\vec{c}_3 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{c}.\vec{b}_2}{|\vec{b}_2|^2}\vec{b}_2$, $\vec{c}_4 = \vec{a} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}$. Then which of the following is a set of mutually orthogonal vectors is

(a) $\{\vec{a}, \vec{b}_1, \vec{c}_1\}$ (b) $\{\vec{a}, \vec{b}_1, \vec{c}_2\}$

(c) $\{\vec{a}, \vec{b}_2, \vec{c}_3\}$ (d) $\{\vec{a}, \vec{b}_2, \vec{c}_4\}$

Sol. **b**

Clearly $\vec{a}.\vec{b}_1 = 0$

$\Rightarrow$ $\vec{a}.\vec{c}_2 = 0$

$\Rightarrow$ $\vec{b}_1.\vec{c}_2 = 0$