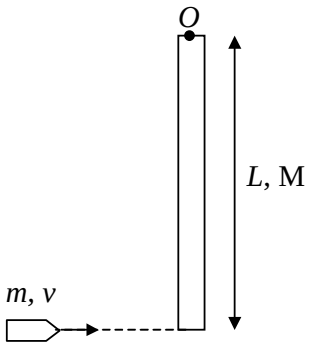
SOLUTIONS TO IIT - JEE, 2005 (MAINS)
MEMORY BASED QUESTIONS
PHYSICS

1. A rod of length L and mass M is hinged at point O. A small bullet of mass m hits the rod as shown in the figure. The bullet gets embedded in the rod. Find angular velocity of the system just after impact.



[2]

Sol. Applying conservation of angular momentum about hinge O,

$$mvL = \left(mL^2 + \frac{ML^2}{3} \right)\omega$$

$$\omega = \frac{3mvL}{3mL^2 + ML^2}$$

$$\omega = \frac{3mv}{L(3m + M)}$$

2. An observer standing on a railway crossing receives frequencies of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train. [The speed of the sound in air is 300 m/s]

[2]

Sol. $$f' = \frac{f_o(v \pm v_o)}{(v \mp v_s)} = \frac{f_o(v)}{v \mp v_s}$$

Approaches: $2.2 = \dfrac{f_o(300)}{300 - v_s}$ (1)

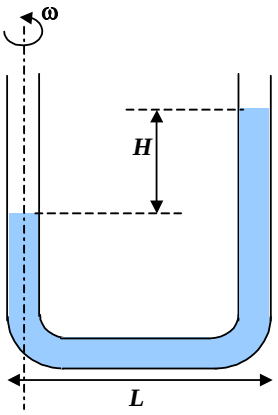
Recedes: $1.8 = \dfrac{f_o(300)}{300 + v_s}$ (2)

Dividing (1) by (2),

$$\frac{2.2}{1.8} = \frac{300 + v_s}{300 - v_s} \quad \Rightarrow \quad \frac{4}{0.4} = \frac{600}{2v_s}$$

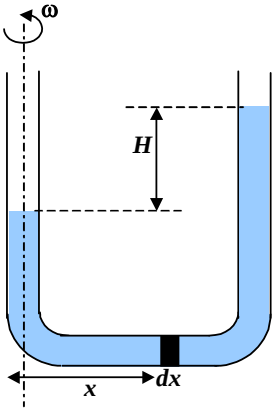
$\Rightarrow \quad v_s = 30$ m/s

3. A U shaped tube contains a liquid of density ρ and it is rotated about the line as shown in the figure. Find the difference in the levels of liquid column.



[2]

Sol. For circular motion of small element dx



$$A(dP) = \rho(Adx)x\omega^2$$

$$\int_{P_1}^{P_2} dP = \rho\omega^2 \int_0^L x\,dx$$

$$(P_2 - P_1) = \frac{\rho\omega^2 L^2}{2}$$

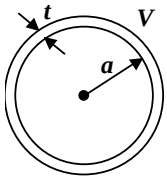
$$\rho g H = \frac{\rho\omega^2 L^2}{2}$$

$$H = \frac{\omega^2 L^2}{2g}$$

4. A conducting bubble of radius a, thickness t ($t << a$) has potential V. Now the bubble collapses into a droplet. Find the potential of the droplet.

[2]

Sol. Potential at surface of bubble,



$$V = \frac{Kq}{a}$$

$$q = \frac{Va}{K} \qquad (1)$$

Since bubble collapses into droplet of radius R. So

$$(4\pi a^2)t = \frac{4}{3}\pi R^3$$

$$R = (3a^2 t)^{1/3}$$

Now potential of droplet

$$V' = \frac{Kq}{R} = \frac{K.(Va)}{K.(3a^2 t)^{1/3}}$$

$$V' = \frac{Va}{(3a^2 t)^{1/3}} = V\left(\frac{a}{3t}\right)^{1/3}$$

5. The potential energy of a particle varies as

$V(x) = E_o \qquad 0 \le x \le 1$

$= 0 \qquad x > 1$

For $0 \le x \le 1$, de Broglie wavelength is λ_1 and for $x > 1$ the de Broglie wavelength is λ_2. Total energy of the particle is $2E_o$. Find λ_1/λ_2.

[2]

Sol. For $0 \le x \le 1$, KE = E_o

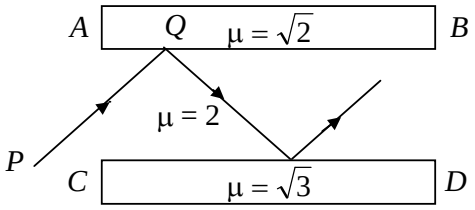
$$\therefore \quad \lambda_1 = \frac{h}{\sqrt{2mE_o}}$$

For $x > 1$, KE = $2E_o$

$$\therefore \quad \lambda_2 = \frac{h}{\sqrt{2m.2E_o}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{h\ \sqrt{2m.2E_o}}{\sqrt{2mE_o}\ h} = \sqrt{2}$$

6. *AB* and *CD* are two slabs. The medium between the slabs has refractive index 2. Find the minimum angle of incidence at *Q* so that the ray is totally reflected by both the slabs.

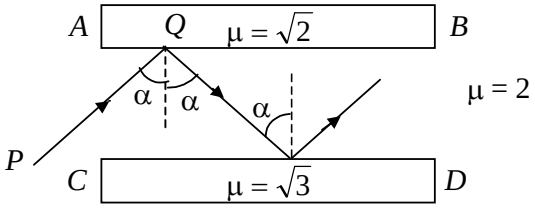


[2]

Sol. For first slab

$$\theta_{C_1} = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\theta_{C_1} = 45^o \qquad (1)$$



For second slab

$$\theta_{C_2} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta_{C_2} = 60^o \qquad (2)$$

For total internal reflection on both the surfaces minimum angle of incidence.

$\alpha = 60^O$

7. The edge of a cube is measured using a vernier calliper. [9 divisions of the main scale is equal to 10 divisions of vernier scale and 1 main scale division is 1 mm]. The main scale division reading is 10 and 1 division of vernier scale was found to be coinciding with the main scale. The mass of the cube is 2.736 gm. Calculate the density in gm/cm^3 upto correct significant figures.

[2]

Sol. 9 MSD = 10 VSD

1 MSD = 1 mm

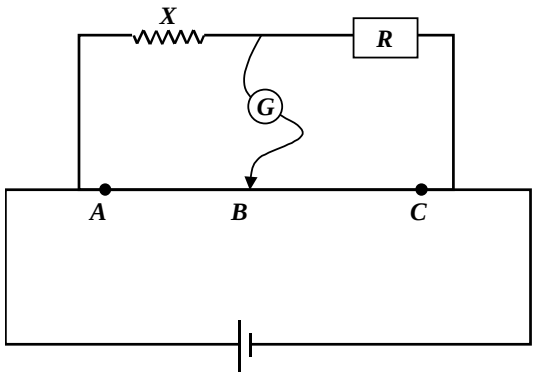
L.C. = 1 MSD - 1 VSD = 1 mm - $\frac{9}{10}$ mm = $\frac{1}{10}$ mm

Measured reading of edge = MSR + VSR × (LC) = 10 + 1 × $\frac{1}{10}$ = 10.1 mm

Volume of cube = $(10.1)^3$ = 1030.301 mm^3 = 1.030301 cm^3

Density of the cube = $\frac{2.736}{1.030301}$ = 2.66 gm/cm^3 .

8. R_1, R_2, R_3 are different values of R. A, B, C are the null points obtained corresponding to R_1, R_2 and R_3 respectively. For which resistor, the value of X will be the most accurate and why?



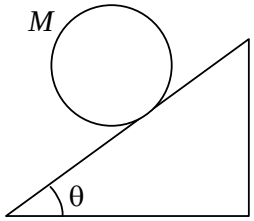
[2]

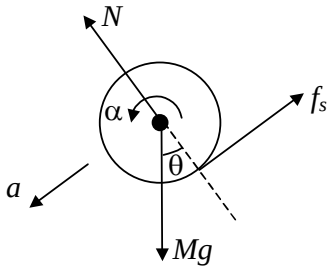
Sol. Slide wire bridge is most sensitive when the resistance of all the four arms of bridge is same.
Hence null point B will give most accurate answer.

9. A solid cylinder rolls without slipping on an inclined plane inclined at an angle θ. Find the linear acceleration of the cylinder. Mass of the cylinder is M.

[4]

Sol.





$Mg\sin\theta - f_s = Ma$ (1)

$f_s.R = I\alpha \quad \Rightarrow \quad f_s.R = \frac{MR^2}{2}\alpha$

$\Rightarrow \quad f_s = \frac{MR}{2}\alpha$ (2)

$a = \alpha R$ (3)

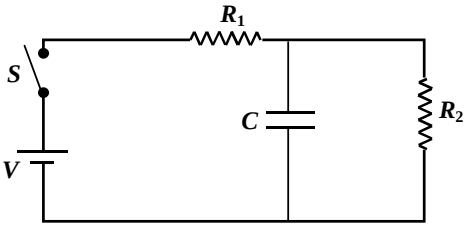
From (3) in (2), $f_s = \frac{Ma}{2}$ (4)

From (4) in (1), $Mg\sin\theta - \frac{Ma}{2} = Ma$

$\Rightarrow \quad Mg\sin\theta = \frac{3}{2}Ma$

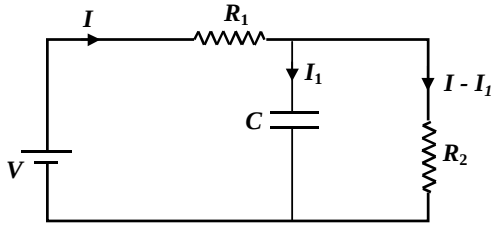
$\Rightarrow \quad a = \frac{2}{3}g\sin\theta$

10. At $t = 0$, switch S is closed. The charge on the capacitor is varying with time as $Q = Q_o\left(1 - e^{-\alpha t}\right)$. Obtain the value of Q_o and α in the given circuit parameters.



[4]

Sol.



Applying KVL,

$$V - IR_1 - \frac{q}{C} = 0$$

$$\frac{q}{C} - (I - I_1)R_2 = 0$$

Also $I_1 = \frac{dq}{dt}$

Solving the above equations,

$$q = \frac{VCR_2}{(R_1 + R_2)}\left(1 - e^{-\frac{(R_1+R_2)t}{R_1R_2C}}\right)$$

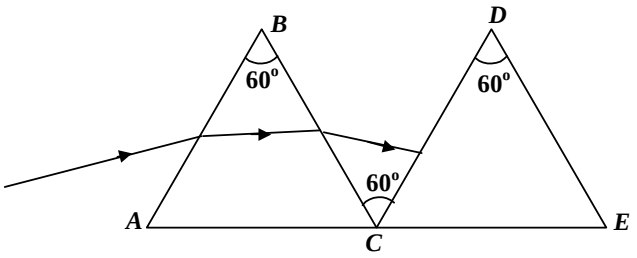
$\therefore \quad \alpha = \frac{(R_1 + R_2)}{R_1R_2\,C}$

$$Q_o = \frac{CVR_2}{R_1 + R_2}$$

11. A ray of light is incident on a prism *ABC* of refractive index $\sqrt{3}$ as shown in figure.

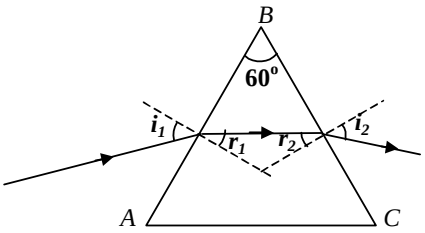
(a) Find the angle of incidence for which the deviation of light ray by the prism *ABC* is minimum.

(b) By what angle the second prism must be rotated, so that the final ray suffer net minimum deviation.



[4]

Sol.(a)



For minimum deviation

$i_1 = i_2 = i$ and $r_1 = r_2 = r$

and $r_1 + r_2 = A$

$r = A/2 = 30^o$

Using Snell's law,

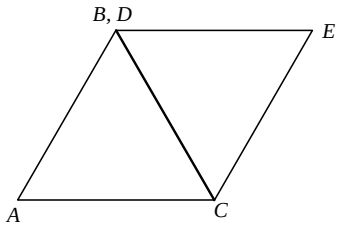
$$\frac{\sin i_1}{\sin r_1} = \mu$$

$$\sin i_1 = \sqrt{3} \times \sin 30^o$$

$$\sin i_1 = \frac{\sqrt{3}}{2}$$

$$i_1 = 60^o$$

(b) When the prism is rotated anticlockwise by 60^o the combined prism will become a slab as shown in the figure.



In this case the net deviation by the prism will be zero as the deviation by the two prism will be equal and opposite.

12. A harmonically moving transverse wave on a string has a maximum particle velocity and acceleration of 3 m/s and 90 m/s^2 respectively. Velocity of the wave is 20 m/s. Find the waveform.

[4]

Sol. $v = 20$ m/s

$y = A \sin(kx \pm \omega t)$ (harmonic wave)

$v_{max} = A\omega = 3$ (1)

$a_{max} = A\omega^2 = 90$ (2)

From (1) and (2), $\omega = 90/3 = 30$ rad/s

$$\therefore \quad A = \frac{3}{\omega} = \frac{3}{30} = 0.1 \text{ m}$$

$$k = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2}$$

$$\therefore \quad y = 0.1 \sin\left[\frac{3}{2}x \pm 30t\right] \text{ m}$$

13. X-rays are incident on a target metal atom having 30 neutrons. The ratio of atomic radius of the target atom and ${}_2^4He$ is $(14)^{1/3}$.

(a) Find the atomic number of target atom.

(b) Find the frequency of K_α line emitted by this metal.

$R = 1.1 \times 10^7$ m^{-1}, $c = 3 \times 10^8$ m/s.

[4]

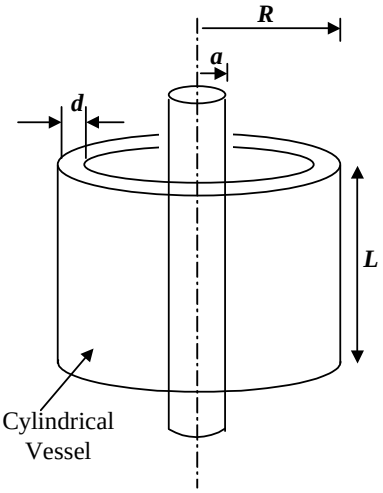
Sol. $$\frac{r_2}{r_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{A_2}{4}\right)^{1/3} \quad \Rightarrow \quad (14)^{1/3} = \left(\frac{A_2}{4}\right)^{1/3} \quad \Rightarrow \quad A_2 = 56$$

Atomic number of second element $Z_2 = A_2 - N_2 = 56 - 30 = 26$

$$f_{K_\alpha} = \frac{3RC}{4}(Z-1)^2 = \frac{3}{4} \times 1.1 \times 10^7 \times 3 \times 10^8 \times (26 - 1)^2$$

$$= \frac{9.9}{4} \times 10^{15} \times 625 = 1546.875 \times 10^{15} \text{ Hz} = 1.55 \times 10^{18} \text{ Hz}$$

14. A long solenoid has n turns per unit length and radius a. A current $I = I_o \sin \omega t$ flows through it. A cylindrical vessel of radius R, length L, thickness d ($d << R$) and resistivity ρ is kept coaxially as shown in the figure. Find the induced current in the outer cylindrical vessel.



[4]

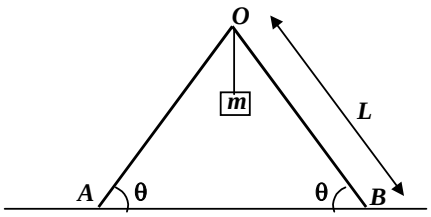
Sol. $$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt} = -A\frac{d}{dt}(\mu_o nI)$$

$$= -\mu_o nAI_o \omega\cos\omega t = \mu_o n\pi a^2 I_o \omega\cos\omega t$$

Resistance $R = \rho\frac{l}{S} = \rho\frac{2\pi R}{Ld}$

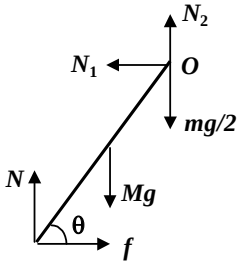
Induced current $I = \frac{\varepsilon}{R} = \frac{\mu_o n\pi a^2 I_o \omega\cos\omega t}{\rho 2\pi R/Ld} = \frac{Ld\mu_o n a^2 I_o \omega\cos\omega t}{2\rho R}$

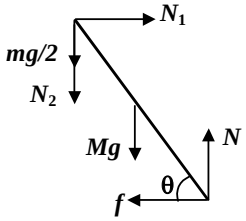
15. Two identical ladders are arranged as shown in the figure. Mass of each ladder is M and length L. The system is in equilibrium. Find direction and magnitude of friction force acting at A or B.



[4]

Sol. Drawing the FBD of both rods





As the rods are in equilibrium

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\tau_{net} = 0$

For right rod

$$N_2 + \frac{mg}{2} + Mg = N \qquad (1)$$

$$N_1 = f \qquad (2)$$

For left rod

$$N_2 + N = Mg + \frac{mg}{2} \qquad (3)$$

$$N_1 = f$$

From (1) and (3)

$$\Rightarrow \quad N_2 = 0 \quad \Rightarrow \quad N = Mg + \frac{mg}{2}$$

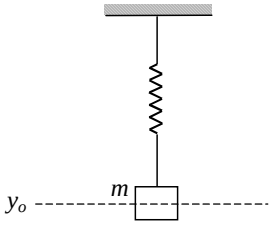
Balancing torque about O,

$$Mg\frac{L}{2}\cos\theta + fL\sin\theta = NL\cos\theta$$

$$\frac{Mg\cos\theta}{2} + f\sin\theta = \left(Mg + \frac{mg}{2}\right)\cos\theta$$

$$f = \left(\frac{Mg}{2} + \frac{mg}{2}\right)\cot\theta \qquad \Rightarrow \qquad f = \left(\frac{M+m}{2}\right)g\cot\theta$$

16. A mass m is undergoing SHM in the vertical direction about the mean position y_o with amplitude A and angular frequency ω. At a distance y from the mean position, the mass detaches from the spring. Assume that the spring contracts and does not obstruct the motion of m. Find the distance y^* (measured from the mean position) such that the height h attained by the block is maximum. ($A\omega^2 > g$)



[4]

Sol. When the mass detaches from the spring at a distance y above the mean position its velocity $v = \omega\sqrt{A^2 - y^2}$.

After detachment the block will have a downward uniform acceleration g.

Total height attained $h = \frac{v^2}{2g} + y = \frac{\omega^2\left(A^2 - y^2\right)}{2g} + y$

$$\Rightarrow \qquad h = \frac{\omega^2 A^2 - \omega^2 y^2 + 2gy}{2g} \qquad \Rightarrow \qquad \frac{dh}{dy} = \frac{-2\omega^2 y + 2g}{2g}$$

For h to be maximum $\frac{dh}{dy} = 0$

$$\Rightarrow \qquad -2\omega^2 y^* + 2g = 0 \qquad \Rightarrow \qquad y^* = \frac{g}{\omega^2}$$

17. A metal of mass 1 kg at constant atmospheric pressure and at initial temperature 20° C is given a heat of 20,000 J. Find the following:

(a) Change in temperature

(b) Work done

(c) Change in internal energy.

Given: Specific heat 400 J/kg/°C.

Coefficient of cubical expansion $\gamma = 9 \times 10^{-5}/^oC$

Density $\rho = 9000$ kg/m^3, atmospheric pressure = 10^5 N/m^2.

[6]

Sol. $\Delta Q = ms\Delta T + = 400\Delta T$

$\Rightarrow \quad \Delta T = 50\ ^oC$

$\Rightarrow \quad T_2 = T_1 + \Delta T = 20 + 50 = 70\ ^oC$

Change in volume $\Delta V = V_1\gamma\Delta T = \frac{1}{9000} \times 9 \times 10^{-5} \times 50 = 5 \times 10^{-7}$

Work done $W = P\Delta V = 10^5 \times 5 \times 10^{-7} = 0.05$ J

Change in internal energy $\Delta U = \Delta Q - W = 20{,}000 - 0.05 = 19{,}999.95$ J

18. A moving coil galvanometer experience Torque $= ki$, where i is current
If N coils of area A each and moment of inertia I is kept in magnetic field B.

(a) find k in terms of given parameters,

(b) if for current i deflection is $\frac{\pi}{2}$, find out torsional constant of spring,

(c) if a charge Q is passed suddenly through the galvanometer find out maximum angle of deflection.

[6]

Sol. (a) $\vec{\tau} = \vec{M} \times \vec{B}, \quad M = NiA$

$\tau = ki = BiNA$

$\therefore \quad k = BNA$

(b) $\tau = K\theta = BiNA$ (K is torsional constant)

$$K = \frac{2BiNA}{\pi} \quad \left[\because \theta = \frac{\pi}{2}\right]$$

(c) $\tau = BiNA$

$$\int_0^t \tau dt = \int_0^t BNA\, idt$$

$$I\omega == BNAQ$$

$$\omega = \frac{BANQ}{I}$$

$\frac{1}{2} I\omega^2 = \frac{1}{2} K\theta^2$ [At maximum deflection whole K.E. will convert in to potential energy of spring]

$$\theta = \sqrt{\frac{I\omega^2}{K}} = \frac{BANQ}{I}\sqrt{\frac{I\pi}{2BiNA}}$$

$$\theta_{max} = Q\sqrt{\frac{BNA\pi}{2I}}.$$

Note: All these questions of IIT-JEE Mains 2005 are based on the memory of the select PIE students who appeared in the examination. PIE Education does not take any responsibility for any sort of discrepancy whatsoever.